DPP – 3 (Vectors)

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Q 1. Find position vector of point A(2,-1,3) and its magnitude:

(a) $\vec{A} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}, |\vec{A}| = \sqrt{14}$

(b) $\vec{A} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}, |\vec{A}| = \sqrt{24}$

(c) $\vec{A} = 2\hat{\imath} - \hat{\jmath} - 3\hat{k}, |\vec{A}| = \sqrt{14}$

(d) None of these

Q 2. If the dot product of two non-zero vectors $\vec{V_1}$ and $\vec{V_2}$ is zero, what does that tell us?

(a) $\vec{V_1} = \vec{V_2}$ (b) $\vec{V_1}$ is parallel to $\vec{V_2}$

(c) $\vec{V_1}$ is perpendicular to $\vec{V_2}$ (d) $\vec{V_1}$ is a component of $\vec{V_2}$

Q 3. Find the dot product of the pair of vectors $\vec{A} = 4\hat{\imath} + \hat{\jmath}, \vec{B} = -\hat{\imath} - \hat{\jmath}$?

(a) 5 (b) 4 (c) -5 (d) -4

Q 4. If a vector $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$, is perpendicular to the vector $4\hat{\imath} - 4\hat{\jmath} + \alpha\hat{k}$. Then the value of α is:

(a) -4 (b) $\frac{1}{4}$ (c) 4 (d) $-\frac{1}{4}$

Q 5. The vector sum of two forces is perpendicular to their vector differences. In that case, the forces

(a) Are not equal to each other in magnitude.

(b) Are equal to each other in magnitude.

(c) Are equal to each other.

(d) Cannot be predicted.

Q 6. Let $\vec{A} = \hat{\imath} + \hat{\jmath}$ and, $\vec{B} = 2\hat{\imath} - \hat{\jmath}$. The magnitude of a coplanar vector $\vec{C}$ such that $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{C} = \vec{A}\cdot\vec{C}$, is given by:

(a) $\sqrt{\frac{10}{9}}$ (b) $\sqrt{\frac{5}{9}}$ (c) $\sqrt{\frac{12}{9}}$ (d) $\sqrt{\frac{9}{12}}$

Q 7. The angle between two vectors $-2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ and $\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$ is:

(a) 0^o (b) 90^o (c) 180^o (d) None of these

Q 8. Given vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath}$, and vector $\vec{b} = \hat{\imath} + \hat{\jmath}$. What is the vector component of $\vec{a}$ in the direction of $\vec{b}$:

(a) $\frac{5}{2}\hat{\imath} + \frac{5}{2}\hat{\jmath}$ (b) $5\hat{\imath} + 5\hat{\jmath}$

(c) $2\hat{\imath} + 2\hat{\jmath}$ (d) None of these

Q 9. Find the angle between $\vec{A} = 4\hat{\imath} + \hat{\jmath} + 3\hat{k}$ and $\vec{B} = \hat{\imath} + 3\hat{\jmath} + 4\hat{k}$:

(a) $\cos^{-1}\frac{26}{19}$ (b) $\cos^{-1}\frac{19}{26}$

(c) $\cos^{-1}\frac{21}{26}$ (d) None of these

Q 10. The position vectors of points A, B, C and D are $\vec{A} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$, $\vec{B} = 4\hat{\imath} + 5\hat{\jmath} + 6\hat{k}$, $\vec{C} = 7\hat{\imath} + 9\hat{\jmath} + 3\hat{k}$ and $\vec{D} = 4\hat{\imath} + 6\hat{\jmath}$ then the displacement vectors AB and CD are?

(a) Perpendicular (b) Parallel

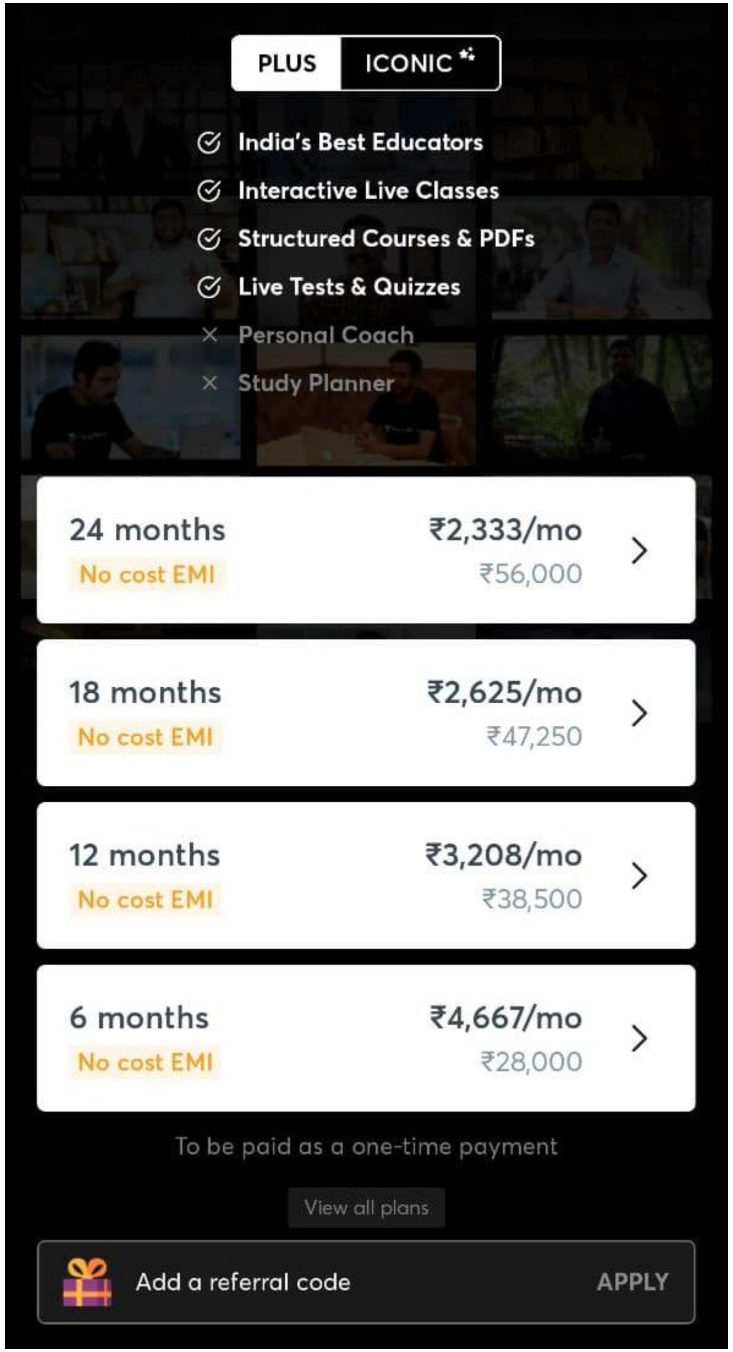
(c) Antiparallel (d) Inclined at an angle of 60^0

Q 11. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 7, |\vec{b}| = 5, |\vec{c}| = 3$. then the angle between c and b is:

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) π

Answer Key

Q.1 a	**Q.2 c**	**Q.3 c**	**Q.4 a**	**Q.5 b**
Q.6 b	**Q.7 b**	**Q.8 a**	**Q.9 b**	**Q.10 c**
Q.11 a				



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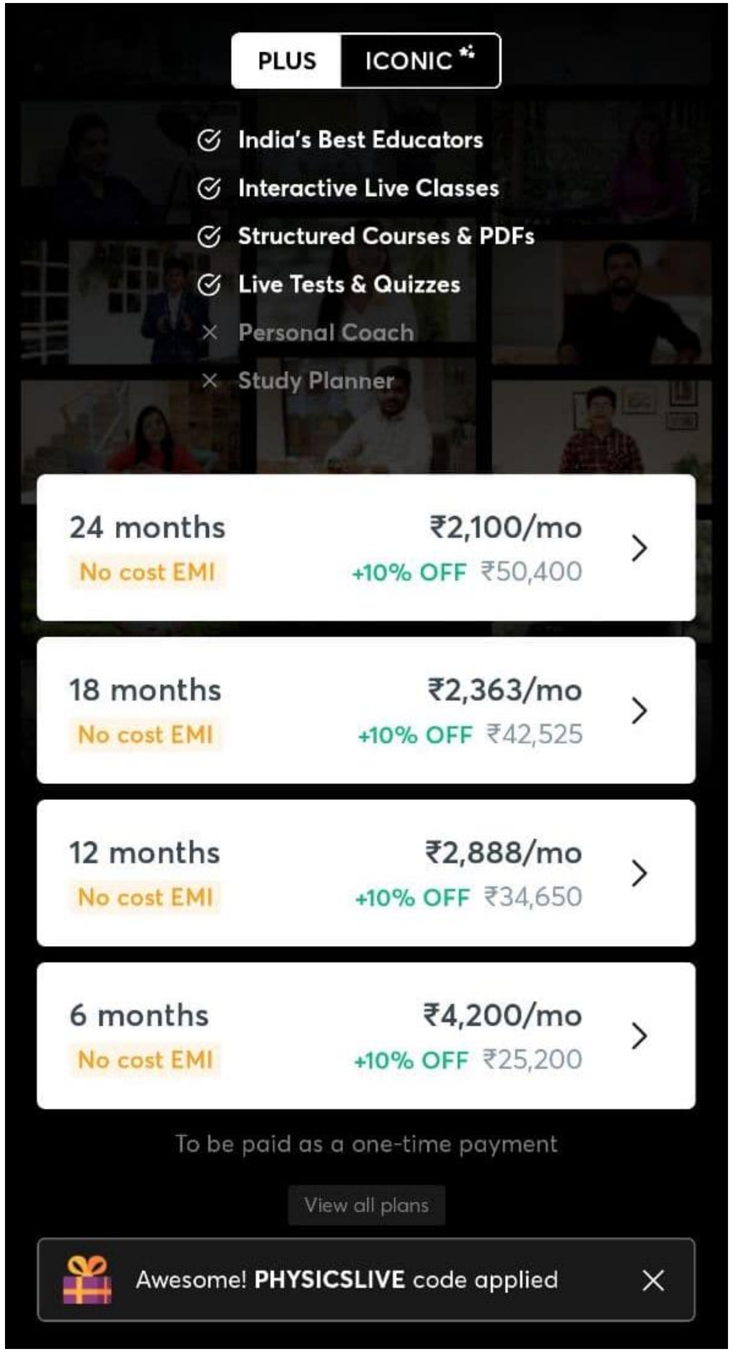

Written Solution

DPP-3 Vectors (Dot product)

By Physicsaholics Team

Solution.1

$A(2, -1, 3)$

$$\vec{A} = 2\hat{i} - \hat{j} + 3\hat{k}$$

$$|\vec{A}| = \sqrt{x^2 + y^2 + z^2}$$

$$|\vec{A}| = \sqrt{(2)^2 + (-1)^2 + (3)^2}$$

$$|\vec{A}| = \sqrt{14}$$

$$\therefore \vec{A} = 2\hat{i} - \hat{j} + 3\hat{k} \;;\; |\vec{A}| = \sqrt{14}$$

Ans.a

Solution.2

if $\vec{V_1} \cdot \vec{V_2} = 0$

$|\vec{V_1}|\,|\vec{V_2}| \cos\theta = 0$

$\cos\theta = 0$

$\boxed{\theta = 90^\circ}$

$\Rightarrow$ $\vec{V_1}$ & $\vec{V_2}$ are perpendicular to each other.

Ans.c

Solution.3

$$\vec{A} = 4\hat{i} + \hat{j}$$

$$\vec{B} = -\hat{i} - \hat{j}$$

$$\vec{A}\cdot\vec{B} = (4\hat{i} + \hat{j})\cdot(-\hat{i} - \hat{j})$$

$$= (4\hat{i}\cdot(-\hat{i})) + (4\hat{i}\cdot(-\hat{j})) + (\hat{j}\cdot(-\hat{i})) + (\hat{j}\cdot(-\hat{j}))$$

$$= -4 + 0 + 0 - 1$$

$$\vec{A}\cdot\vec{B} = -4 - 1$$

$$\boxed{\vec{A}\cdot\vec{B} = -5}$$

Ans.c

Solution.4

if two vector are perpendicular

⇒ so, their dot product will be zero

$\therefore (2\hat{i} - \hat{j} + 3\hat{k}) \cdot (4\hat{i} - 4\hat{j} + \alpha\hat{k}) = 0$

$(2 \times 4) + (-1 \times -4) + (3 \times \alpha) = 0$

$8 + 4 + 3\alpha = 0 \Rightarrow \alpha = \frac{-12}{3}$

$\boxed{\alpha = 4}$

Ans.a

Solution.5

given; $(\vec{a}+\vec{b}) \perp (\vec{a}-\vec{b})$

$\therefore (\vec{a}+\vec{b}) \cdot (\vec{a}-\vec{b}) = 0$

$\vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{b} = 0$

$\because \vec{a}\cdot\vec{a} = a^2$

$\vec{b}\cdot\vec{b} = b^2$

& $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

$\therefore a^2 - ab + ab - b^2 = 0$

$a^2 - b^2 = 0$

$a^2 = b^2$

$\because a^2 = |\vec{a}|^2$ & $b^2 = |\vec{b}|^2$

$\therefore a = b$

$|\vec{a}| = |\vec{b}|$

$\therefore \vec{a}$ & $\vec{b}$ are equal in magnitude.

Ans.b

Solution.6

$\vec{A} = \hat{i} + \hat{j}$, $\vec{B} = 2\hat{i} - \hat{j}$

let $\vec{C} = x\hat{i} + y\hat{j}$

$\therefore \vec{A}\cdot\vec{B} = (\hat{i}+\hat{j})\cdot(2\hat{i}-\hat{j})$

$= 2 - 1$

$\boxed{\vec{A}\cdot\vec{B} = 1}$

$\vec{B}\cdot\vec{C} = (\hat{i}+\hat{j})\cdot(x\hat{i}+y\hat{j}) = x + y$

& $\vec{A}\cdot\vec{C} = (2\hat{i}-\hat{j})\cdot(x\hat{i}+y\hat{j}) = 2x - y$

$\therefore \vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{C} = \vec{A}\cdot\vec{C}$

$1 = x + y = 2x - y$

$x + y = 1$, & $2x - y = y + x$

$\Rightarrow \boxed{y = 1 - x}$ $\Rightarrow 2x - x = 2y$

$\boxed{y = \frac{x}{2}}$

$\Rightarrow 1 - x = \frac{x}{2} \Rightarrow 1 = x + \frac{x}{2} = \frac{3x}{2}$

$\boxed{x = \frac{2}{3}}$

$y = 1 - x = 1 - \frac{2}{3} = \frac{1}{3}$

$\boxed{y = \frac{1}{3}}$

$\vec{C} = \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j}$

$|\vec{C}| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2}$

$|\vec{C}| = \sqrt{\frac{4}{9} + \frac{1}{9}}$

$\boxed{|\vec{C}| = \sqrt{5/9}}$

Ans.b

Solution.7

$\vec{a} = -2\hat{i} + 3\hat{j} + \hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$

for angle between $\vec{a}$ & $\vec{b}$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$|\vec{a}| = \sqrt{(-2)^2 + (3)^2 + (1)^2} = \sqrt{14}$

$|\vec{b}| = \sqrt{(1)^2 + (2)^2 + (4)^2} = \sqrt{21}$

$\vec{a}\cdot\vec{b} = (-2\hat{i} + 3\hat{j} + \hat{k})\cdot(\hat{i} + 2\hat{j} - 4\hat{k})$

$\vec{a}\cdot\vec{b} = -2 + 6 - 4 = 0$

$$\cos\theta = \frac{0}{\sqrt{14}\cdot\sqrt{21}}$$

$\cos\theta = 0$

$\boxed{\theta = 90^\circ}$

Ans.b

Solution.8

$\vec{a} = 2\hat{i} + 3\hat{j}$

$\vec{b} = \hat{i} + \hat{j}$

Component of $\vec{a}$ along $\vec{b}$ is :-

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\hat{b}$$

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{\hat{i}+\hat{j}}{\sqrt{1^2+1^2}} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$$

$$\vec{a}\cdot\vec{b} = (2\hat{i}+3\hat{j})\cdot(\hat{i}+\hat{j})$$

$$= 2+3 = 5$$

$$\text{Component} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\hat{b}$$

$$= \frac{5}{\sqrt{2}} \times \left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$$

$$= \frac{5}{2}\hat{i} + \frac{5}{2}\hat{j}$$

Ans.a

Solution.9

$\vec{A} = 4\hat{i} + \hat{j} + 3\hat{k}$; $\vec{B} = \hat{i} + 3\hat{j} + 4\hat{k}$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{(4\hat{i}+\hat{j}+3\hat{k})\cdot(\hat{i}+3\hat{j}+4\hat{k})}{(\sqrt{4^2+1^2+3^2})(\sqrt{1^2+3^2+4^2})}$$

$$\cos\theta = \frac{4+3+12}{(\sqrt{26})(\sqrt{26})} = \frac{19}{26}$$

$$\theta = \cos^{-1}\left(\frac{19}{26}\right)$$

Ans.b

Solution.10

$\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{B} = 4\hat{i} + 5\hat{j} + 6\hat{k}$

$\vec{C} = 7\hat{i} + 9\hat{j} + 3\hat{k}$

$\vec{D} = 4\hat{i} + 6\hat{j}$

$\vec{AB} = \vec{B} - \vec{A}$

$\boxed{\vec{AB} = \hat{i} + \hat{j} + \hat{k}}$

$\vec{CD} = \vec{D} - \vec{C}$

$\boxed{\vec{CD} = -3\hat{i} - 3\hat{j} - 3\hat{k}}$

$\vec{CD} = -3(\hat{i} + \hat{j} + \hat{k})$

$\vec{CD} = -3(\vec{AB})$

$\therefore$ angle between $\vec{CD}$ & $\vec{AB}$ is "180°"

So, $\vec{CD}$ & $\vec{AB}$ are antiparallel.

Ans.c

Solution.11

$$\vec{a}+\vec{b}+\vec{c}=0$$

$$\vec{a}+\vec{b}=-\vec{c}$$

$$\vec{b}+\vec{c}=-\vec{a}$$

$$|\vec{b}+\vec{c}|=|\vec{a}|$$

$$|\vec{b}+\vec{c}|^2=|\vec{a}|^2$$

$$\because (\vec{b}+\vec{c})\cdot(\vec{b}+\vec{c})=|\vec{b}+\vec{c}|^2$$

$$\therefore (\vec{b}+\vec{c})\cdot(\vec{b}+\vec{c})=|\vec{a}|^2$$

$$|\vec{b}|^2+|\vec{c}|^2+2|\vec{b}||\vec{c}|\cos\theta=|\vec{a}|^2$$

where; θ = angle between $\vec{b}$ & $\vec{c}$

$|a|=7,\ |b|=5,\ |c|=3$

$$\Rightarrow (5)^2+(3)^2+2(5)(3)\cos\theta=(7)^2$$

$$\cos\theta=\frac{49-25-9}{30}=\frac{15}{30}=\frac{1}{2}$$

$$\boxed{\theta=\frac{\pi}{3}}$$

Ans.a

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Chalo Niklo